

<b>Ninth Semester (5<sup>th</sup> year of MSc)</b>					
1	Measure Theory and Integration	MA501	3-1-0	4	70
2	Advanced Mathematical Modelling and Simulation	MA503	3-0-2	4	85
3	Probability and Statistics-II	MA531	3-1-0	4	70
4	Financial Mathematics	MA532	3-1-0	4	70
5	Elective (Core Elective)	MA5AA	3-X-X	4	55/70/85
			<b>Total</b>	<b>20</b>	<b>350-380</b>
<b>Tenth Semester (5<sup>th</sup> year of MSc)</b>					
1	Dissertation**	MAP10	0-0-40	20	800 (40x 20)
			<b>Total</b>	<b>20</b>	<b>800</b>

<b>M.Sc. V (Mathematics), Semester – IX</b> <b>MEASURE THEORY AND</b> <b>INTEGRATION MA 501</b>	<b>Scheme</b>	<b>L</b>	<b>T</b>	<b>P</b>	<b>Credit</b>
		<b>3</b>	<b>1</b>	<b>0</b>	<b>04</b>

<b>1.</b>	<b>Course Outcomes (COs):</b> <b>At the end of the course, the students will be able to</b>
CO1	Explain the fundamentals of measure theory.
CO2	demonstrate the Lebesgue measure with their properties and characterization.
CO3	apply the concept of measure theory in integration.
CO4	extend the theorems related to measure theory and integration.
CO5	utilize the concept of integration in different spaces.

<b>2.</b>	<b>Syllabus</b>	
	<b>INTRODUCTION</b>	<b>(09 Hours)</b>
	Introduction, Extended real numbers, Algebra and sigma-algebra of subsets of a set, Sigma algebra generated by a Class, Monotone Class, Set functions, The length function, and its properties. Countably additive set functions on intervals, Uniqueness problem for measure. Extension of measure, Outer measure and its properties, A Measurable set.	
	<b>LEBESGUE MEASURE</b>	<b>(11 Hours)</b>
	Lebesgue measure and its properties, Characterization of Lebesgue measurable sets, Measurable functions, Properties of measurable functions, Measurable functions on measure spaces, Integral of non-negative simple measurable functions, Properties of non-negative simple measurable functions, Monotone convergence theorem and Fatou's Lemma.	
	<b>INTEGRATION OF FUNCTION (REAL VARIABLE)</b>	<b>(10 Hours)</b>
	Properties of integrable functions and dominated convergence theorem, Dominated convergence, Theorem and applications, Lebesgue integral and its properties.	
	<b>MEASURE AND INTEGRATION</b>	<b>(10 Hours)</b>
	An introduction to product measure, Construction of product measures, Computation of product measure, Integration on product spaces, Fubini's theorems, Lebesgue measure and integral in Euclidean space, Properties of Lebesgue measure in Euclidean space, Lebesgue integral in Euclidean space.	
	<b>THE RIEMANN-STEILTJES INTEGRAL</b>	<b>(05 Hours)</b>

	Definitions and existence of the integral, Conditions of integrability, The integral as a limit of sum, Some important theorems.	
	<b>Tutorials will be based on the coverage of the above topics separately.</b>	<b>(15 Hours)</b>
	<b>(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)</b>	

<b>3.</b>	<b>Tutorials will be based on</b>
1	Introduction.
2	Lebesgue Measure.
3	Integrable Functions.
4	Product Measure.
5	Lebesgue Measure and Integral in Euclidean Space.
6	The Riemann-Steiltjes Integral.

<b>4.</b>	<b>Books Recommended</b>
1	G. De Barra, Measure Theory and Integration, New Age International Publisher, New Delhi, 2017.
2	P. K. Jain, Measure Theory and Integration, New Age International Publisher, New Delhi, 2019.
3	J. L. Doob, Measure Theory, Springer, New York, 2012.
4	S. C. Malik and S. Arora, Mathematical Analysis, 6th Edition, New Age International (P) Limited, New Delhi, 2021.
5	Tao, Terence, <i>An Introduction to Measure Theory</i> , 1st Edition, Cambridge University Press, 2011.

<b>5.</b>	<b>Additional Books</b>
1	I. K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, New Delhi, 2007.

<b>M.Sc. V (Mathematics), Semester – IX</b> <b>ADVANCED MATHEMATICAL MODELLING</b> <b>AND SIMULATION</b> <b>MA 503</b>	<b>Scheme</b>	<b>L</b>	<b>T</b>	<b>P</b>	<b>Credit</b>
		<b>3</b>	<b>0</b>	<b>2</b>	<b>04</b>

<b>1.</b>	<b>Course Outcomes (COs):</b> <b>At the end of the course, the students will be able to</b>
CO1	explain the concept of mathematical modelling & simulation.
CO2	formulate the real-world problem into Mathematical form.
CO3	analyze the mathematical model.
CO4	apply simulation in the analysis of real-world problems.
CO5	develop a computer code for the simulation of a system.

<b>2.</b>	<b>Syllabus</b>	
	<b>INTRODUCTION TO MATHEMATICAL MODELLING</b>	<b>(11 Hours)</b>
	Introduction to mathematical modelling, Real-world problems, Identification of parameters, Significant parameters, Importance of parameters, Reduction of an open problem to a closed form, Conversion of a real problem into a mathematical problem, Quest for a mathematical technique for the solution, Importance of numerical techniques, Physical interpretation of solution, Types of mathematical models, Characteristics of mathematical models, Framework of mathematical models, Validation of mathematical model, Advantage and disadvantage of the mathematical model.	
	<b>MATHEMATICAL MODELS</b>	<b>(12 Hours)</b>
	Models based on system of algebraic equations, ODE based simple modelling, Population dynamics modelling, multi-compartmental modelling, Detection of diabetic model, Technological innovation model, Heat and mass transport Models, Heat conduction and diffusion Problems.	
	<b>INTRODUCTION TO SIMULATION</b>	<b>(11 Hours)</b>
	Introduction to simulation, Types of simulation, Simulation methodology, Random number generation, Monte-Carlo simulation, Simulation of continuous system, Discrete event simulation, Design of experiments, Validation.	
	<b>SOME CASE STUDIES</b>	<b>(11 Hours)</b>
	Simulation of queuing system, Simulation of inventory control, Simulation of forecasting, Simulation of maintenance problem, Simulation of network problems, Simulation of regression analysis, Simulation of linear programming problems.	
	<b>Practicals will be based on the coverage of the above topics separately.</b>	<b>(30 Hours)</b>
	<b>(Total Contact periods / Hrs.: 45 Hrs. + 30 Hrs. = 75 Hrs.)</b>	

<b>3.</b>	<b>Practicals</b>
1	Radioactive decay and Newton's law of Colling Model with their simulation
2	Population dynamics model and their simulation
3	Single Compartment modelling and their simulation
4	Multi-compartment modelling and their simulation
5	Technological innovation model and their simulation
6	Detection of Diabetic model and their simulation
7	Analytical solution of PDE with MATLAB and their simulation (Heat Equation)
8	Analytical solution of PDE with MATLAB and their simulation (Wave Equation)
9	Simulation for Forecasting and PERT Network
10	Operation research based Simple simulation problems

<b>4.</b>	<b>Books Recommended</b>
1	J. N. Kapoor, Mathematical Modeling, New Age International(p) Limited, New Delhi,2023.
2	B. Barnes and G. R. Fulford, Mathematical Modelling with Case Studies, Using Maple and MATLAB, 3 <sup>rd</sup> Edition, CRC Press,Boca Raton, 2019.
3	Clegg, Andrew (Ed.), <i>Mathematical Modeling: Principles and Applications</i> , Springer, Switzerland,2018
4	N. Deo, System Simulation with Digital Computer, PHI New Delhi, India,2011.
5	Sokolowski, John A. and Banks, Catherine M., <i>Modeling and Simulation Fundamentals</i> , Wiley,Hoboken, 2010.

<b>5.</b>	<b>Additional books</b>
1	F. L. Seervance, System Modeling and Simulation: An Introduction, John Wiley, Hoboken,2001.
2	J. Caldwell, K. S. Ng Douglas and J. Caldwell, Mathematical Modeling: Case Studies and Projects (Texts in the Mathematical Sciences), Springer Netherlands, 2004.

<b>M.Sc. V (Mathematics), Semester – IX PROBABILITY AND STATISTICS-II MA 531</b>	<b>Scheme</b>	<b>L</b>	<b>T</b>	<b>P</b>	<b>Credit</b>
		<b>3</b>	<b>1</b>	<b>0</b>	<b>04</b>

<b>1.</b>	<b>Course Outcomes (COs): At the end of the course, the students will be able to</b>
CO1	learn the various technique of data reduction.
CO2	apply the knowledge of testing of hypothesis in real-world problems.
CO3	elaborate nonparametric statistics and analysis of categorical data.
CO4	utilize statistical techniques for quality control.
CO5	discuss various stochastic processes and their applications, and apply the knowledge of the Markov Chain to industrial requirements.

<b>2.</b>	<b>Syllabus</b>	
	<b>PRINCIPLES OF DATA REDUCTION</b>	<b>(08 Hours)</b>
	Sufficiency, Minimal sufficiency, Factorization theorem, Completeness, Completeness of exponential families, Ancillary statistic, Basu's theorem, and its applications, uniformly minimum variance unbiased estimation, Rao-Blackwell theorem, Lehmann-Scheffe theorem, Cramer-Rao inequality, Consistent estimators, Convergence in probability, convergence in distribution.	
	<b>HYPOTHESIS TESTING</b>	<b>(08 Hours)</b>
	Tests of Hypothesis and tests of significance, Critical region and Level of significance, Errors in the testing of hypothesis, most powerful tests, Monotone likelihood ratio (MLR) property, uniformly most powerful tests, uniformly most powerful tests for families having MLR property, Uniformly most powerful unbiased tests, Uniformly most powerful unbiased tests for exponential families, Likelihood ratio tests, Large sample tests.	
	<b>PARAMETRIC TEST</b>	<b>(08 Hours)</b>
	Z-test, t-test, F-test, ANOVA for randomized block design, Factorial Experiments	
	<b>NON-PARAMETRIC STATISTICS</b>	<b>(06 Hours)</b>
	Chi-square test, Contingency tables: A two-way classification, ways of comparing proportions, Measures of associations, Kolmogorov-Smirnov test, Sign test, Wilcoxon signed rank test, Mann-Whitney U-test, The Kruskal-Wallis H-test.	

	<b>STATISTICAL QUALITY CONTROL</b>	<b>(07 Hours)</b>
	Objectives of Quality Control, Causes of Variation in quality, Techniques of SQC, Control charts for Variables (X-charts & R-chart S-chart & $\sigma$ -chart), 6 $\sigma$ concept, Control charts for Attributes (p-charts, np- chats, C-charts), Statistical process control, Terms used in sampling Inspection plans.	
	<b>STOCHASTIC PROCESS</b>	<b>(08 Hours)</b>
	Markov chains with finite and countable state space, Classification of states, Limiting behaviour of n-step transition probabilities, Stationary distribution, Brownian motion, and its basic properties.	
	<b>Tutorials will be based on the coverage of the above topics separately.</b>	<b>(15 Hours)</b>
	<b>(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)</b>	

<b>3.</b>	<b>Tutorials will be based on</b>
1	Principles of Data Reduction.
2	Hypothesis Testing.
3	Parametric Test.
4	Non-Parametric Statistics.
5	Statistical Quality Control.
6	The Stochastic Process.

<b>4.</b>	<b>Books Recommended</b>
1	W. Mendenhall, R. J. Beaver and B. M. Beaver, Introduction to Probability & Statistics, 15 <sup>th</sup> Edition, Cengage Learning, Boston, 2020.
2	S. Ross, A First Course in Probability, 10 <sup>th</sup> Edition, Pearson Education India, Delhi, 2022.
3	D. C. Montgomery and G. C. Runger, Applied Statistics and Probability for Engineers, 7 <sup>th</sup> Edition, Wiley, Hoboken, 2019.
4	T. Veerarajan, Probability and Statistics and Random Processes, 3 <sup>rd</sup> Edition, Tata McGraw-Hill, New Delhi, India, 2017.
5	Casella, George and Berger, Roger L., <i>Statistical Inference</i> , 2nd Edition, Duxbury Press, 2002. Scheduled for new edition, Chapman & Hall/CRC, Boca Raton, May 23, 2024.

<b>M.Sc. V (Mathematics), Semester – IX</b> <b>Financial Mathematics</b> <b>MA532</b>	<b>Scheme</b>	<b>L</b>	<b>T</b>	<b>P</b>	<b>Credit</b>
		<b>3</b>	<b>1</b>	<b>0</b>	<b>04</b>

<b>1.</b>	<b>Course Outcomes (COs):</b> <b>At the end of the course, the students will be able to</b>
CO1	relate the basic concepts of Probability in financial mathematics
CO2	demonstrate the concepts relating to functions and annuities
CO3	explain the Arbitrage theorem and Black-Scholes formula
CO4	employ methods related to these concepts in a variety of financial applications
CO5	apply logical thinking to problem related to call and put options and valuing of investment.

<b>2.</b>	<b><u>Syllabus</u></b>	
	<b>INTRODUCTION</b>	<b>(07 Hours)</b>
	Stochastic Processes, Poisson Process, Brownian Motion, Martingales Present Value Analysis.	
	<b>INTERESTS RATES AND PRESENT VALUE ANALYSIS</b>	<b>(07 Hours)</b>
	Interest rates, Present value analysis, Rate of return, Continuously varying Interest rates.	
	<b>THE ARBITRAGE THEOREM</b>	<b>(07 Hours)</b>
	Market Model Specification problems, Arbitrage Theorem, Multi-period binomial Model, Proof of the Arbitrage Theorem.	
	<b>THE BLACK-SCHOLES FORMULA</b>	<b>(08 Hours)</b>
	The Black-Scholes formula, Properties of the Black–Scholes Option Cost, The Delta Hedging Arbitrage Strategy.	
	<b>ADDITIONAL RESULTS ON OPTION</b>	<b>(06 Hours)</b>
	Call Options on Dividend-Paying Securities, Pricing American Put Options, Estimating the Volatility Parameter.	
	<b>VALUING BY EXPECTED UTILITY</b>	<b>(10 Hours)</b>
	Valuing investments by expected utility, Portfolio selection problem, Capital Assets Pricing model, Rates of return, Single period and geometric Brownian motion, Mean variance analysis of risk - neutral priced call options, Autoregressive models and mean regression, Other pricing options and applications.	
	<b>Tutorials will be based on the coverage of the above topics separately.</b>	<b>(15 Hours)</b>
	<b>(Total Contact Periods/ Hrs.: 45 Hrs. + 15 Hrs.= 60 Hrs.)</b>	

<b>3.</b>	<b>Tutorials</b>
1	Stochastic Processes, Poisson Process
2	Brownian Motion, Martingales Present Value Analysis.
3	Present value analysis, Rate of return, continuously varying Interest rates
4	Market Model Specification problems, Arbitrage Theorem, Multi-period binomial Model
5	Properties of the Black–Scholes Option Cost, The Delta Hedging Arbitrage Strategy
6	Dividend-Paying Securities, Pricing American Put Options,
7	Estimating the Volatility Parameter
8	Portfolio selection problem, Capital Assets Pricing model
9	Rates of return, Single period and geometric Brownian motion
10	Analysis of risk - neutral priced call options, Autoregressive models and mean regression

<b>4.</b>	<b><u>Books Recommended:</u></b>
1.	S. M. Ross, An Introduction to Mathematical Finance, Cambridge University Press,Cambridge, 2011.
2	Benninga, Simon, <i>Financial Modeling</i> , 5th Edition, MIT Press, 2014.
3.	McDonald, Robert L., <i>Introduction to Derivatives and Risk Management</i> , 9th Edition, Cengage Learning, Boston,2013.
4.	Joshi, M. S., <i>The Concepts and Practice of Mathematical Finance</i> , 2nd Edition, Cambridge University Press,Cambridge, 2011.
5.	M. Capiński and T. Zastawniak, <i>Mathematics for Finance: An Introduction to Financial Engineering</i> , 2nd Edition Springer, Berlin, Germany,2011.

<b>5.</b>	<b><u>Additional Books :</u></b>
1.	M. S. Joshi, <i>The Concepts and Practice of Mathematical Finance</i> , 2nd Edition, Cambridge University Press, Cambridge, UK,2008.
2.	A. J. Prakash, R. M. Bear, K. Dandapani, G. L. Ghai, T. E. Pactwa and A. M. Parhizgari, <i>The Return Generating Models in Global Finance</i> , Pergamon Press, Oxford,UK,2007.
3.	P. Wilmott, <i>Derivatives: The Theory and Practice of Financial Engineering (Frontiers in Finance Series)</i> , John Wiley & Sons, Hoboken,1998.

<b>M.Sc. V (Mathematics), Semester –IX ADVANCE OPERATIONS RESEARCH MA 551</b>	<b>Scheme</b>	<b>L</b>	<b>T</b>	<b>P</b>	<b>Credit</b>
		<b>3</b>	<b>1</b>	<b>0</b>	<b>04</b>

<b>1.</b>	<b>Course Outcomes (COs): At the end of the course, the students will be able to</b>
CO1	explain the importance of operations research in real-life problems.
CO2	apply basic concepts of Mathematics to formulate and solve OR problems.
CO3	incorporate chance factor and calculate project completion time in PERT & CPM.
CO4	interpret multi-stage decision processes through dynamic programming
CO5	solve NLPP using different methods.

<b>2.</b>	<b>Syllabus</b>	
	<b>INTRODUCTION</b>	<b>(04 Hours)</b>
	Nature and scope of Operations Research, Convex sets and convex functions and their properties.	
	<b>INVENTORY MODELS</b>	<b>(07 Hours)</b>
	Inventory control -Deterministic including price breaks and multi-item with constraints, Probabilistic (with and without lead time).	
	<b>QUEUING THEORY</b>	<b>(07 Hours)</b>
	Basic Structures of queuing models, Poisson queues- M/M/1, M/M/C for finite and infinite queue length, non-Poisson queue -M/G/1, Machine Maintenance (steady state).	
	<b>PERT AND CPM</b>	<b>(07 Hours)</b>
	Introduction, Basic difference between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM Network components and precedence relationships, Critical path analysis, Probability in PERT analysis, Project Time-Cost, Trade-off, updating of the project, Resource allocation-resource smoothing and resource-leveling.	
	<b>DYNAMIC PROGRAMMING</b>	<b>(07 Hours)</b>
	Introduction, Nature of dynamic programming, Deterministic processes, non-sequential discrete optimization, Allocation problems, Assortment problems, Sequential discrete optimization, Long-term	

	planning problem, Multi-stage decision process, Application of Dynamic Programming in production scheduling and routing problems.	
	<b>NONLINEAR PROGRAMMING</b>	<b>(07 Hours)</b>
	Quadratic Programming, Duality theory, Search techniques - one variable (Fibonacci, Golden Section method) and several variables (Conjugate Gradient, Newton's method).	
	<b>GEOMETRIC PROGRAMMING</b>	<b>(06 Hours)</b>
	Introduction, Posynomial, Arithmetic-Geometric inequality, Geometric programming (both unconstrained and constrained).	
	<b>Tutorials will be based on the coverage of the above topics separately.</b>	<b>(15 Hours)</b>
	<b>(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)</b>	

<b>3.</b>	<b>Tutorials will be based on</b>
1	Introduction and Inventory Models.
2	Queuing Theory.
3	PERT and CPM.
4	Dynamic Programming.
5	Nonlinear Programming.
6	Geometric Programming.

<b>4.</b>	<b>Books Recommended</b>
1	F. S. Hiller and G. J. Lieberman, Introduction to Operations Research, 10 <sup>th</sup> Edition, Tata McGraw-Hill, New Delhi, 2017.
2	K. Swarup, P. K. Gupta and M. Mohan, Operations Research, 19 <sup>th</sup> Edition, S. Chand & Sons, New Delhi, 2017.
3	H. A. Taha, Operations Research: An Introduction, 10 <sup>th</sup> Edition, Pearson, New Delhi, 2019.
4	J. K. Sharma, Operations Research: Theory and Applications, 6 <sup>th</sup> Edition, Trinity Press, New Delhi, 2017.
5	M. S. Bazarrá, H. D. Sherali and C. M. Shetty, Nonlinear Programming: Theory and Algorithms, 3 <sup>rd</sup> Edition, John Wiley & Sons, New Jersey, 2016.

<b>M.Sc. V (Mathematics) Semester – IX FLUID DYNAMICS IN POROUS MEDIA MA552</b>	<b>Scheme</b>	<b>L</b>	<b>T</b>	<b>P</b>	<b>Credit</b>
		<b>3</b>	<b>1</b>	<b>0</b>	<b>04</b>

<b>1.</b>	<b>Course Outcomes (COs): At the end of the course, the students will be able to</b>
CO1	explain the fundamental properties of porous media
CO2	elaborate filtration theory
CO3	elaborate the permeability and hydraulic conductivity
CO4	implement the appropriate method for the groundwater problem
CO5	use continuity equation for groundwater flow.

<b>2.</b>	<b><u>Syllabus</u></b>	
	<b>FUNDAMENTAL PROPERTIES OF POROUS MEDIA</b>	<b>(12 Hours)</b>
	Porosity: general aspects and definition, Determination of porosity, Compressibility of porous media, Classification of porosity, Capillary properties, Saturation, Wettability, Classification of reservoir rocks based on wettability, Measurement of wettability, Contact angle measurement, Amott method, Capillary pressure, Definition, Measurement of the capillary pressure in a porous medium, Method of centrifuge, Mercury injection (Purcell method), The Leverett function, Pore size distribution, Vertical equilibrium, Permeability, Darcy's law, Definition and units of permeability, Measurements of permeability, Klinkenberg effect, Analogies between the laws of Darcy, Ohm and Fourier, Filtration velocity, Quadratic equation of filtration, Relative permeabilities, Definition of relative permeability, Definitions of end-point saturations, Relative permeability measurements, The HASSLER method, PENN-STATE-method, Welge-method, Saturation distribution, and relative permeability	
	<b>INTRODUCTION TO GROUNDWATER</b>	<b>(07 Hours)</b>
	Introduction, Types of springs, Infiltration gallery, Karsez, distribution of water on earth, Groundwater resources of India, Geological formations, Properties of aquifers: void ratio, Specific retention, Specific yield, Methods of determining specific yield, Pumping method, The porosity, Specific yield, and specific retention of different formations	
	<b>PERMEABILITY AND HYDRAULIC CONDUCTIVITY IN SOIL FORMATION</b>	<b>(10 Hours)</b>
	Permeability and hydraulic conductivity, Hydraulic conductivity or permeability coefficient, The effect of porosity, Pore space geometry, Submergence, Tortuosity of soil pores, Entrapped air, Measurement of hydraulic conductivity, Constant head method, Variable head method, Field methods, Below the water table field methods, Above the water field method, Intrinsic	

	permeability, Apparent specific yield, Coefficient of storage, Specific storage, Hydraulic resistance, Leakage factor, Distribution of subsurface water, Zone of aeration, Soil water zone, Intermediate zone, Capillary fringe, Important features of the capillary fringe zone, Zone of saturation, Soil moisture, Groundwater flow potential, Measurement of groundwater, Conjunctive use of groundwater.	
	<b>CONTINUITY EQUATIONS FOR GROUNDWATER FLOW</b>	<b>(16 Hours)</b>
	Introduction, Three-dimensional continuity equation for groundwater flow, the Continuity equation for homogeneous and isotropic formation (Medium), Confined and unconfined aquifer, General continuity equation in Cartesian coordinates, General continuity equation in polar coordinates, Continuity equation for confined aquifer with leakage from top and bottom, Dupuit-Forchheimer theory for unconfined aquifer with recharge, Flow through an unconfined aquifer.	
	<b>Tutorials will be based on the coverage of the above topics separately.</b>	<b>(15 Hours)</b>
	<b>(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)</b>	

<b>3.</b>	<b>Tutorials will be based on</b>
1	The Fundamental Properties of Porous Media.
2	The Introduction to Ground Water.
3	Permeability and Hydraulic Conductivity in Soil Formation.
4	Continuity Equations for Groundwater Flow.

<b>4.</b>	<b>Books Recommended</b>
1	J. Bear, Dynamics of Fluids in Porous Media, Dover Publications, New York, 2013.
2	V. C. Agarwal, Groundwater Hydrology, PHI Learning Private Limited, New Delhi, 2012.
4	K. Vafai, Handbook of Porous Media, 3 <sup>rd</sup> Edition, CRC Press, Florida, 2015.
5	M.B. Allen, The Mathematics of Fluid Flow Through Porous Media, John Wiley & Sons, Inc., New Jersey, 2021.

M.Sc. V (Mathematics), Semester –IX ADVANCED NUMERICAL ANALYSIS MA 553	Scheme	L	T	P	Credit
		3	1	0	04

1.	<b>Course Outcomes (COs):</b> <b>At the end of the course, the students will be able to</b>
CO1	solve the initial value problems using multistep methods.
CO2	develop a finite difference scheme for ODEs and PDEs.
CO3	analyze the stability and convergence of difference schemes.
CO4	determine the solution for ODEs and PDEs using finite difference methods.
CO5	assess the suitability and effectiveness of finite difference schemes.

2.	<b>Syllabus</b>	
	<b>REVIEW OF THE SYSTEM OF LINEAR EQUATIONS</b>	<b>(05 Hours)</b>
	Condition number and ill-conditioned systems. Matrix and vector norms. Error bounds, tridiagonal and pentagonal system of equations.	
	<b>INITIAL VALUE PROBLEMS IN ODES</b>	<b>(08 Hours)</b>
	Review single step methods, explicit multistep methods, implicit multistep methods, Predictor and corrector methods, Stability and convergence analysis.	
	<b>FINITE DIFFERENCE METHODS FOR BVPS IN ODES</b>	<b>(08 Hours)</b>
	Review on numerical approximation to derivatives, Approximation of boundary conditions of different kinds, Solution of linear and non-linear boundary value problems, and Convergence of difference schemes.	
	<b>FINITE DIFFERENCE METHODS FOR HYPERBOLIC PDES</b>	<b>(08 Hours)</b>
	Difference schemes in one space dimension with constant coefficient, Convergence and consistency, Stability, The Lax-Richtmyer theorem, The CFL condition, Analysis of difference schemes: Fourier and Von Neumann analysis, Stability condition, Order of accuracy of difference schemes	
	<b>FINITE DIFFERENCE METHODS FOR PARABOLIC PDES</b>	<b>(08 Hours)</b>
	Explicit and implicit Difference schemes in one, two, and three space dimensions. Two level and multilevel schemes, Solution of convection-diffusion equation. Consistency, stability, and convergence of difference scheme.	

	<b>FINITE DIFFERENCE METHODS FOR ELLIPTIC PDES</b>	<b>(08 Hours)</b>
	Approximation to $\nabla^2$ and $\nabla^4$ . Five-point and nine-point approximation for Laplace and Poisson equations, Dirichlet problem, ADI method, Neumann Problem, Mixed boundary value problems.	
	<b>Tutorials will be based on the coverage of the above topics separately.</b>	<b>(15 Hours)</b>
	<b>(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)</b>	

<b>3.</b>	<b>Tutorials will be based on</b>
1	Review of the System of Linear Equations.
2	Initial Value Problems in Odes.
3	Finite Difference Methods for BVPS In ODES.
4	Finite Difference Methods for Hyperbolic PDEs.
5	Finite Difference Methods for Parabolic PDEs.
6	Finite Difference Methods for Elliptic PDEs.

<b>4.</b>	<b>Books Recommended</b>
1	M. K. Jain, Numerical Solution of Differential Equations, New Age Publication, New Delhi, 2018.
2	Ames, William F., <i>Numerical Methods for Partial Differential Equations</i> , Dover Publications, New York 2014.
3	LeVeque, Randall J., <i>Finite Difference Methods for Ordinary and Partial Differential Equations: Numerical Methods for Steady-State and Time-Dependent Problems</i> , SIAM, Philadelphia, 2012.
4	Chapra, S. C., <i>Numerical Methods for Partial Differential Equations</i> , McGraw-Hill, New York, 2012.
5	Anderson, John D., <i>Finite Difference Methods for Differential Equations</i> , Elsevier, Oxford, UK, 2011.

<b>5.</b>	<b>Additional Books</b>
1	J. C. Strikwerda, Finite Difference Schemes for Partial Differential Equations, 2nd Edition, SIAM, 2007.
2	R. J. LeVeque, Finite Difference Methods for Ordinary and Partial Differential Equations: Steady State and Time-Dependent Problems, SIAM, 2007.
3	G. D. Smith, Numerical Solutions of Partial Differential Equations, 3rd Edition, Clarendon Press, Oxford, 1986.

M.Sc. V (Mathematics), Semester – IX LINEAR OPERATORS AND APPROXIMATION THEORY MA 554	Scheme	L	T	P	Credit
		3	1	0	04

1.	<b>Course Outcomes (COs):</b> <b>At the end of the course, the students will be able to</b>
CO1	demonstrate the basics of approximation theory and basic problems.
CO2	evaluate the order of approximation of functions by means of polynomials and polynomial operators.
CO3	apply notions and theorems concerning the trigonometric and polynomial approximation.
CO4	demonstrate the concepts of linear continuous operators.
CO5	construct the Fourier series approximation of a periodic function.

2.	<b>Syllabus</b>	
	<b>LINEAR FUNCTIONALS AND OPERATORS</b>	<b>(10 Hours)</b>
	Linear positive functional, Linear positive operators, Approximation of functions by algebraic polynomials, Approximation of functions by trigonometric polynomials, Conditions for convergence of a sequence of linear positive operators.	
	<b>ORDER OF APPROXIMATION OF FUNCTIONS BY POLYNOMIALS</b>	<b>(11 Hours)</b>
	Polynomials which deviate the least from functions, Modulus of continuity, General methods of summation of Fourier series, Order of approximation of functions by means of trigonometric polynomials, Order of approximation of functions by means of algebraic polynomials. Order of growth of derivatives of polynomials and trigonometric polynomials.	
	<b>ORDER OF APPROXIMATION OF FUNCTIONS BY MEANS OF LINEAR POSITIVE POLYNOMIAL OPERATORS</b>	<b>(10 Hours)</b>
	Order of approximation of functions by means of linear positive functional, Order of approximation of functions by means of Fejer operators, Order of approximation of functions by means of Bernstein polynomials, Order of approximation of functions by means of linear positive polynomial operators.	
	<b>LINEAR CONTINUOUS POLYNOMIAL OPERATORS</b>	<b>(05 Hours)</b>
	Linear continuous operators, Auxiliary relations, non-uniformly convergent sequence of linear continuous polynomial operators, Valle e-Poussin operators.	
	<b>FOURIER SERIES</b>	<b>(09 Hours)</b>
	The Fourier series, Uniform convergence of Fourier series, Mean convergence of Fourier series, Local convergence, Estimate of the deviation of partial sums of a Fourier series, Example of a continuous function not expandable in a Fourier series, Convergence of sequence of linear positive polynomial operators, General methods of summation of Fourier series	
	<b>Tutorials will be based on the coverage of the above topics separately.</b>	<b>(15 Hours)</b>
	<b>(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)</b>	

<b>3.</b>	<b>Tutorials will be based on</b>
1	Linear Functionals and Operators.
2	The Order of Approximation of Functions by Polynomials.
3	The Order of Approximation of Functions by Means of Linear Positive Polynomial Operators.
4	Linear Continuous Polynomial Operators.
5	The Fourier Series.

<b>4.</b>	<b>Books Recommended</b>
1	P. P. Korovkin, Linear Operators and Approximation Theory, Hindustan Publishing Corporation (India), 2017.
2	Trefethen, Lloyd N., <i>Approximation Theory and Approximation Practice</i> , SIAM, 2013.
3	Dahmen, Wolfgang, <i>Constructive Approximation</i> , Springer, Berlin, Germany 2013.
4	Berberian, S. K., <i>Approximation Theory and Functional Analysis</i> , Springer, Berlin, Germany, 2012.
5	H. M. Mhaskar, and D. V. Pai, Fundamentals of Approximation Theory, Narosa Publishing House, New Delhi, India, 2011.

<b>5.</b>	<b>Additional Books</b>
1	E. W. Cheney, Introduction to Approximation Theory, 2 <sup>nd</sup> Revised Edition, AMS Chelsea Publishing Co., Providence Co., 1982
2	I. P. Natanson, Constructive Function Theory Volume-I, Fredrick Ungar Publishing Co., New York, 1964.
3	A. F. Timan, Theory of Approximation of Functions of a Real Variable, Dover Publication Inc., Mineola, New York, 1994.