	Seventh Semester (4th year of MSc)				
1	Functional Analysis	MA401	3-1-0	4	70
2	Abstract Algebra	MA403	3-1-0	4	70
3	Fluid Dynamics	MA405	3-1-0	4	70
4	Optimization Techniques	MA431	3-1-0	4	70
5	Elective (Core Elective)	MA4AA	3-X-X	3/4	55/70/85
6	MOOC Course*	MA457	3-0-0/ 3-1-0	3/4	55/70/85
			Total	**23-24**	**390-450**
7	Mini Project-II Preliminary Part-I Vocational Training / Professional Experience (Optional) (mandatory for exit)	MAV07 / MAP07	0-0-10	5	200 (20 X 10)
	Eighth Semester (4th year of MSc)				
1	Topology	MA402	3-1-0	4	70
2	Higher Transcendental Functions	MA404	3-1-0	4	70
3	Partial Differential Equations	MA406	3-1-0	4	70
4	Calculus of Variations & Integral Equations	MA432	3-1-0	4	70
5	Elective (Core Elective)	MA4CC	3-X-X	4	55/70/85
6	MOOC Course*	MA458	3-0-0/ 3-1-0	3/4	**390-450**
			Total	**23-24**	**335-365**
7	Mini Project-II Preliminary Part-II Vocational Training / Professional Experience (Optional) (mandatory for exit)	MAV08 / MAP08	0-0-10	5	200 (20 X 10)

M.Sc. IV (Mathematics) Semester – VII FUNCTIONAL ANALYSIS MA401	Scheme	L	T	P	Credit
		3	1	0	04

1.	**Course Outcomes (COs):** **At the end of the course, students will be able to**
CO1	demonstrate the concept of normed linear spaces
CO2	explain bounded linear map, their properties and applications
CO3	prove the theorems related to Hilbert space
CO4	discuss the concept of dual spaces and corresponding operator theory
CO5	prove spectral theorem for different operators

2.	**Syllabus**	
	FUNDAMENTALS OF NORMED LINEAR SPACE	**(06 Hours)**
	Normed linear spaces, Finite dimensional spaces, Riesz lemma.	
	BOUNDED LINEAR MAPS ON NORMED LINEAR SPACES	**(10 Hours)**
	Definition and examples, Linear maps on finite dimensional spaces, Operator norm, Banach spaces, Hahn-Banach theorem and its applications, Open mapping and closed graph theorems, Uniform boundedness principle.	
	HILBERT SPACES	**(08 Hours)**
	Inner product spaces, Orthonormal sets, Gram-Schmidt orthogonalization, Bessel's inequality, Orthonormal basis, Separable Hilbert spaces, projection and Riesz representation theorem, Divergence of Fourier series.	
	DUAL SPACES AND ADJOINT OF AN OPERATOR	**(07 Hours)**
	Duals of classical spaces, Weak and weak* convergence, Banach-Alaoglu theorem, Adjoint of an operator.	
	BOUNDED OPERATORS ON HILBERT SPACE	**(08 Hours)**
	Adjoint operator, Normal, Unitary, Self-adjoint operator, Compact operator, Eigenvalues, Eigenvectors, Banach algebras.	
	SPECTRAL THEOREM	**(06 Hours)**
	Spectral theorem for compact self-adjoint operators, Spectral theorem for bounded Self-adjoint operators and unitary operators.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**
	(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)	

3.	**Tutorials will be based on**
1	normed linear spaces.
2	boundedness.
3	Hilbert spaces.
4	dual spaces, adjoint of operators etc.
5	spectrum theorem, and related topics.

4.	**Books Recommended**
1	B. V. Limaye, "Functional Analysis", 3rd Revised Edition, New Age International Private Limited, New Delhi,2014.
2	G. F. Simmons, "Introduction to Topology and Modern Analysis", 1st Edition, McGraw-Hill Education, New York, 2017.
3	W. Rudin, "Functional Analysis", 3rd Edition, McGraw-Hill, New York, 2024.
4	J. B. Conway, "A Course in Functional Analysis," 3rd Edition, Springer-Verlag, New York, 2019.
5	J. T. Oden and L. Demkowicz, Applied Functional Analysis, 2nd Edition, CRC Press, Boca Raton,2018.

5.	**Additional Books:**
1.	E. Kreyszig, "Introductory Functional Analysis with Applications", 1st Edition, John Wiley & Sons, New York, 1989.

M.Sc. IV (Mathematics) Semester – VII ABSTRACT ALGEBRA MA403	Scheme	L	T	P	Credit
		3	1	0	04

1.	**Course Outcomes (COs):** **At the end of the course, students will be able to**
CO1	explain abstract algebra's axiomatic theories.
CO2	Strengthen mathematical reasoning abilities by engaging in in-depth analysis and rigorous proof techniques within the realm of advanced group theory.
CO3	comprehend fundamental concepts of ring theory and their significance in contemporary mathematics.
CO4	apply appropriate techniques to prove key results and solve algebraic problems.
CO5	explore field extensions and grasp the fundamentals of Galois theory's theorems.

2.	**Syllabus**	
	ADVANCED GROUP THEORY	**(16 Hours)**
	Conjugacy Classes, Theorems on Finite Groups, Class Equation, Sylow's Theorems, Normal and Subnormal Series, Composition Series, Solvable Groups and Theorems, Jordan-Holder Theorem, Nilpotent Groups.	
	RING THEORY	**(12 Hours)**
	Structures and Properties of Rings, Subrings and Ideals, Sum of Ideals, Product of Ideals, Maximal Ideals, Quotient Rings, Homomorphisms, Polynomial Rings and their Properties, Division Rings, Factorization in Polynomial Rings, Divisibility, Integral Domains, Euclidean Domains, Prime and Irreducible Elements, Principal Ideal Domains and Unique Factorization Domains.	
	FIELD THEORY	**(09 Hours)**
	Fundamentals and Properties of Fields, Skew Fields, Field of Quotients and Embedding Theorems, Eisenstein's Irreducibility Criterion.	
	FIELD EXTENSIONS AND GALOIS THEORY	**(08 Hours)**
	Prime Field, Field Extensions, Splitting Fields, Normal Extensions, Separable and Inseparable Extensions, Automorphisms of Field Extensions, Galois Extensions, Galois Groups, The Fundamental Theorem of Galois Theory.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**
	(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)	

3.	**Tutorials will be based on**
1	the topics: class equation, conjugacy, Sylow’s theorems etc.
2	the topics: normal series, composition series, solvable groups etc.
3	the topics: rings, subrings, ideals, UFD, PID, ED etc.
4	the fundamental properties of finite fields.
5	the topics: field extensions, splitting field, normal extension, separable extension, Galois theory etc.

4.	**Books Recommended**
1	M. Artin, "Algebra, 2nd Edition," Pearson Education India, New Delhi, India,2018.
2	J. A. Gallian, "Contemporary Abstract Algebra,” 10th Edition, Cengage Learning India (P.) Ltd.,New Delhi,India, 2021
3	J. B. Fraleigh, "First Course in Abstract Algebra, “3rd Edition, Narosa Publishing House, New Delhi, 2003.
4	Thomas W. Hungerford, Algebra, 2nd Edition, Springer, Switzerland,2020.
6	D. A. Cox, Galois Theory, 2nd Edition, Wiley,Hoboken, 2012.

4.	**Additional Books:**
1.	D. S. Dummit and R. M. Foote, "Abstract Algebra,” 3rd Edition, John Wiley & Sons Inc., Hoboken,2004.
2.	P. M. Chon, "Algebra, Vols. I, II & III," John Wiley & Sons,New York, 1992.
3.	I. N. Herstein, "Topics in Algebra”, 2nd Edition, Wiley India (P.) Ltd., New Delhi, 2009.

M.Sc. (Mathematics) Semester-VII FLUID DYNAMICS MA 405	Scheme	L	T	P	Credit
		3	1	0	04

1.	**Course Outcomes (COs):** **At the end of the course, students will be able to**
CO1	explain the physical properties of fluid and their consequence on fluid flow
CO2	identify the fundamental kinematics of a fluid element
CO3	analyze practical applications of Bernoulli's equation
CO4	formulate conceptual and analytical models of flow system
CO5	solve problems in fluid dynamics using finite difference methods

2.	**Syllabus**	
	GENERAL INTRODUCTION	**(04 Hours)**
	Introduction to fluid dynamics, Normal and shear stress, The concept of a fluid, Kinds of fluids, Characteristics of fluid, Density, Pressure, Viscosity, Surface tension and compressibility, Different types of flows, Visualization of flows.	
	EQUATIONS OF MOTION	**(06 Hours)**
	Pressure equation (Bernoulli's equation for steady and unsteady motion), Practical applications of Bernoulli's equation to Orifice meter, Pitot-tube, Venturimeter.	
	POTENTIAL FLOW	**(06 Hours)**
	Velocity potential and irrotational flow, Circulation and Kelvin's theorem, Theorem of Blasius, Stream function in two dimensions, Complex velocity potential.	
	GOVERNING EQUATION OF FLUID DYNAMICS	**(12 Hours)**
	Derivation of the Navier-stokes equation, Flow between parallel plates – Couette flow and plane Poiseuille flow. Hagen – Poiseuille flow through pipes, Steady flow through a cylindrical pipe, Steady flow between co-axial circular cylinders.	
	BOUNDARY LAYER FLOW	**(06 Hours)**
	Drag and lift, Prandtl's boundary layer theory, Boundary layer equation, Karman's integral (condition) equation, Flow parallel to a semi-infinite flat plate, Reynold's number, Prandtl number, Nusselt number, Froude number, Eckert number.	
	INTRODUCTION TO COMPUTATIONAL DYNAMICS	**(11 Hours)**

	General introduction and role of computational fluid dynamics in modern fluid dynamics, The method of finite differences, Derivation of elementary finite difference quotients, Basic aspects of finite-difference equations, Errors and an analysis of stability, Explicit finite difference methods; The Lax–Wendroff method, Mac Cormack's method, Stability criterion, Applications of the explicit time-dependent technique, Generalized Crank–Nicholson scheme.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**
	(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)	

3.	**Tutorials will be based on**
1	basics of fluid, shear stress, characteristics of fluid, Density, pressure, viscosity, visualization of flows.
2	Bernoulli's equation for steady and unsteady motion
3	circulation and Kelvin's theorem, Theorem of Blasius, etc.
4	Navier-stokes equation, Hagen – Poiseuille, Steady flow through a cylindrical, etc.
5	drag and lift, Prandtl's boundary layer theory, Karman's integral equation, , Reynold's number, Prandtl number, Nusselt number, etc.

4.	**Books Recommended**
1.	P. K. Kundu, I. M. Cohen and D. Dowling, "Fluid Mechanics", 6th Edition, Elsevier,Amsterdam, 2015.
2.	J. K. Goyal and K. P. Gupta, "Fluid Dynamics and Advanced Hydrodynamics", Pragati Prakashan,Meerut,India,2016.
3.	G. K. Bachelor, "An Introduction to Fluid Dynamics", Cambridge University Press,Cambridge, 2000 (Re-issued in 2012).
4.	Pijush K. Kundu, Ira M. Cohen, and David R. Dowling, Fluid Mechanics, 6th Edition, Academic Press,San Diego,2015.
5.	Clive A. J. Fletcher, Computational Techniques for Fluid Dynamics, 2nd Edition, Springer,Berlin,Germany, 2012.

4.	**Additional Books:**
1.	M. E. O'Neill and F. Chorlton, "Ideal and Incompressible Fluid Dynamics", John Wiley & Sons,New York, 1986.

M.Sc. IV (Mathematics) Semester – VII OPTIMIZATION TECHNIQUES MA 431	Scheme	L	T	P	Credit
		3	1	0	04

1.	Course Outcomes (COs): At the end of the course, students will be able to
CO1	demonstrate the importance of optimization in real life problems.
CO2	formulate the real world problems into optimization problems
CO3	analyze the sensitivity of different components of an LPP towards its solution.
CO4	estimate the solution of different real life problems using the concept of LPP
CO5	explain the basic concept of non-linear optimization

2.	Syllabus	
	LINEAR PROGRAMMING PROBLEMS	**(10 Hours)**
	Introduction, Structure of L.P.P., Formulation of an L.P.P., Graphical Method of solution of L.P.P., Standard form of L.P.P., Simplex Algorithm, Simplex Tableau, Two Phase Method, Big-M Method, Types of Linear Programming solutions, Duality.	
	REVISED SIMPLEX METHOD	**(05 Hours)**
	Revised simplex method (with and without artificial variable), Bounded variable technique, Dual simplex method, Modified dual simplex method.	
	SENSITIVITY ANALYSIS	**(06 Hours)**
	Change in the objective function, Change in the requirement vector, Addition of a variable, Addition of a constraint, Parametric analysis of cost and requirement vector.	
	INTEGER PROGRAMMING PROBLEMS	**(04 Hours)**
	Gomory's cutting plane algorithm, Gomory's mixed integer problem algorithm, A branch and bound algorithm.	
	TRANSPORTATION PROBLEMS	**(05 Hours)**
	Mathematical Model for Transportation Problem, North-West Corner Method, Least Cost Method, Vogel's Approximation Method, Test for optimality, Degeneracy in Transportation Problem, Variations in Transportation Problem.	
	ASSIGNMENT PROBLEMS	**(05 Hours)**
	Mathematical Model for Assignment Problem, Solution Method for Assignment Problem, Variations in Assignment Problem, Traveling Salesman Problem.	

	SEQUENCING PROBLEMS	**(04 Hours)**
	Processing of Jobs through machines: Problems with n jobs two machines, n jobsthree machines and n jobs m machines.	
	INTRODUCTION TO NONLINEAR OPTIMIZATION	**(06 Hours)**
	General NLPP, Formulation, unconstrained and constrained optimization, constrained optimization with equality constraints (Lagrange's theory), constrained optimization with inequality constraints (Kuhn-Tucker conditions).	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**
	(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)	

3.	**Tutorials will be based on**
1	Formation of L.P.P.
2	Graphical Method of solution of L.P.P.
3	Standard form of L.P.P. and Simplex Method.
4	Two Phase Method and Revised Simplex Method.
5	Two Phase Method, Revised Simplex Method and Dual Simplex Method.
5	Sensitivity Analysis and Integer Programming Problems.
7	Transportation, Assignment and Sequencing Problems.
8	Nonlinear Programming Problems

4.	**Books Recommended**
1	K. Swarup, P. K. Gupta and M. Mohan, “Operations Research”, 19th Edition, S. Chand & Sons, New Delhi, 2017.
2	H. A. Taha, “Operations Research: An Introduction”, 10th Edition, Pearson, New Delhi, 2019.
3	J. K. Sharma, “Operations Research: Theory and Applications”, 6th Edition, Trinity Press, New Delhi, 2017.
4	J. C. Pant, “Introduction to Optimization / Operations Research”, 7th Edition, Jain Brothers, New Delhi, 2015.
5	M. S. Bazara. J. J. Jarvis and H. D. Sherali, “Linear Programming and Network Flows”, 4th Edition, John Wiley & Sons, Hoboken,2013.

M.Sc. IV (Mathematics) Semester – VIII TOPOLOGY MA402	Scheme	L	T	P	Credit
		3	1	0	04

1.	Course Outcomes (COs): At the end of the course, students will be able to
CO1	explain the concept of topology and intrinsic properties of topological spaces.
CO2	demonstrate homeomorphism and topological manifold.
CO3	discuss compactness, connectedness and related theorems.
CO4	explain the concept of countability axioms, separability.
CO5	discuss the metrization theorem and its applications in topology.

2.	Syllabus	
	INTRODUCTION	**(14 Hours)**
	Topological spaces, Examples of topological spaces, Subspace topology, Product topology, Metric topology, Order topology, Quotient topology, Bases, Sub bases, Continuous function, Homeomorphism, Topological manifold.	
	COMPACTNESS	**(11 Hours)**
	Compact spaces, Heine-Borel theorem, Local compactness, One-point compactification, Tychonoff's theorem, The Stone-Cech compactification.	
	CONNECTEDNESS	**(07 Hours)**
	Connected spaces, Components and local connectedness.	
	COUNTABILITY & SEPARATION AXIOMS	**(13 Hours)**
	Countability axioms, Separability, i.e., T_0, T_1, T_2 spaces, Regularity, Completed regularity, Normality, Urysohn lemma, Tychonoff embedding and Urysohnmetrization theorem, Tietze extension theorem.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**
	(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)	

3.	**Tutorials will be based on**
1	basics of topology, subspace topology etc.
2	compactness.
3	connectedness.
4	separability.
5	regularity, metrization etc.

4.	**Books Recommended**
1	G. F. Simmons, "Introduction to Topology and Modern Analysis", 4th Edition, McGraw-Hill, New York, 2017.
2	J. L. Kelley, "General Topology," Reprint edition, Dover Publication Inc.,Mineola,New York, 2017.
3	M. A. Armstrong, "Basic Topology," Springer (India), New Delhi,India,2013.
4.	John G. Hocking and Gail S. Young, Topology, Dover Publications, Reprint Edition,Mineola,New York, 2015.
5.	J. R. Munkres, "Topology," 2nd Edition, Pearson Education (India), New Delhi,India,2010.

5.	**Additional Books**
1.	K. D. Joshi, "Introduction to General Topology," New Age International, New Delhi, India,2000.

M.Sc. IV (Mathematics) Semester – VIII HIGHER TRANSCENDENTAL FUNCTIONS MA 404	Scheme	L	T	P	Credit
		3	1	0	04

1.	Course Outcomes (COs): At the end of the course, students will be able to
CO1	explain the fundamentals of higher transcendental functions
CO2	demonstrate the infinite products and their convergence with applications
CO3	analyze with the solution of hypergeometric function and their properties
CO4	develop the generating relations of different functions and polynomials
CO5	explain the concept of q-series with applications

2.	Syllabus	
	INFINITE PRODUCT	**(08 Hours)**
	Definition of infinite product, convergence conditions, The associated series of logarithm, Absolute convergence, Uniform convergence, The Euler or Mascheroni constant, Gamma function, Psi function, Euler product of gamma function, Euler integral for gamma function, Beta function, Legendre duplication formula, Gauss's multiplication theorem.	
	HYPERGEOMETRIC FUNCTION	**(10 Hours)**
	Introduction, Hypergeometric function and generalized hypergeometric function, Integral representation, Differential properties of hypergeometric function, Confluent hypergeometric function and its integral representation.	
	THEORY OF GENERATING FUNCTION	**(09 Hours)**
	Introduction to generating functions, Generating functions of the family of the form $G(2xt - t^2)$, $e^t\phi(t)$etc. with suitable examples (Bessel function, Legendre Polynomial, Hermite polynomial and Laguerre Polynomial), Boas and Buck type, Pure recurrence relations, Appell, Sheffer and 0-type characterizations of polynomial sets.	
	ORTHOGONAL POLYNOMIALS	**(09 Hours)**
	Introduction, The moment functional, Orthogonality, Existence of OPS, The fundamental recurrence formula, Zeros, Gauss quadrature, Kernel polynomials, Symmetric moment functional, Certain related recurrence relations, Orthogonality of Lagguere, Legendre, Hermite and Bessel Functions.	
	BASIC HYPERGEOMETRIC SERIES AND THEIR APPLICATIONS	**(09 Hours)**
	Introduction to basic Hyper geometric series, q-analogue of orthogonal polynomials, q-Gamma and q-Beta functions.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**

(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)

3.	**Tutorials will be based on**
1	convergence conditions, Uniform convergence, Euler constant, Gamma function and Psi function.
2	Euler product, Euler integral for gamma function, Beta function, Legendre duplication formula and Gauss's multiplication theorem.
3	Hypergeometric function, Generalized hypergeometric function, Integral representation, Differential properties of hypergeometric function.
4	Confluent hypergeometric function and its integral representation.
5	Generating functions of the family $G(2xt - t2)$, $et\phi(t)$ etc. with suitable examples (Bessel function, Legendre Polynomial, Hermite polynomial and Laguerre Polynomial)
6	Boas and Buck type, Pure recurrence relations, Appell, Sheffer and 0-type characterizations of polynomial sets.
7	moment functional, orthogonality, recurrence formula, Zeros, Gauss quadrature, Kernel polynomials.
8	Symmetric moment functional, Certain related recurrence relations, Orthogonality of Lagguere, Legendre, Hermite and Bessel Functions.
9	basic hyper geometric series, q-analogue of orthogonal polynomials
10	q-Gamma and q-Beta functions.

4.	**Books Recommended**
1	Richard Beals, Roderick Wong, Special Functions for Scientists and Engineers, Cambridge University Press,Cambridge, 2016.
2	M. Rahman and H. Exton, Basic Hypergeometric Series and Applications, Springer,New York, 2011.
3	John B. Conway, Functions of One Complex Variable, 2nd Edition, Springer, Switzerland,2019.
4	Walter Gautschi, Orthogonal Polynomials: Computation and Approximation, Oxford University Press, Oxford,UK,2010.
5	Nico M. Temme, Special Functions: An Introduction to the Classical Functions of Mathematical Physics, 2nd Edition, Wiley, Hoboken,2014.

M.Sc. IV (Mathematics) Semester – VIII PARTIAL DIFFERENTIAL EQUATIONS MA 406	Scheme	L	T	P	Credit
		3	1	0	04

1.	**Course Outcomes (COs):** **At the end of the course, students will be able to**
CO1	formulate the physical problem into partial differential equations.
CO2	solve first order linear and nonlinear equations
CO3	analyze the existence and uniqueness of solutions
CO4	classify second order equations into three types of PDEs: parabolic, hyperbolic and elliptic
CO5	discuss the formulation and solution of Laplace, wave and heat equation

2.	**Syllabus**	
	INTRODUCTION TO PDE	**(07 Hours)**
	Order and degree of PDE, Types of PDE, Solution of simple PDE, Formation of PDE, Initial and Boundary conditions, Types of solution.	
	FIRST ORDER PDE	**(12 Hours)**
	The method of characteristics, The existence and uniqueness theorem, Cauchy problem, Lagrange's method, Compatible system of first order PDEs , Charpit's method, Jacobi method, Geometrical interpretation and applications of first order PDE.	
	SECOND AND HIGHER ORDER PDE	**(12 Hours)**
	Homogeneous and non-homogeneous PDE of order two and higher with constant coefficient. PDEs reducible to equations with constant coefficients. PDEs of order two with variable coefficients. Classification of PDE, Reduction to canonical or Normal form, Riemann Method, Monges's method.	
	LAPLACE EQUATION	**(05 Hours)**
	Formulation and physical interpretation, Derivation of fundamental solution, Uniqueness of solution, Dirichlet's principle, Method of separation of variables.	
	HEAT EQUATION	**(05 Hours)**
	Formulation and physical interpretation, Derivation of fundamental solution, Uniqueness of solution, Method of separation of variables.	
	WAVE EQUATION	**(04 Hours)**

	Formulation and physical interpretation, D'Alembert's solution, Uniqueness of solution, Method of separation of variables.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**
	(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)	

3.	**Tutorials will be based on**
1	solution of simple PDE, Initial and Boundary conditions, Types of solution, etc.
2	Cauchy problem, Lagrange's method, Compatibility, Charpit's method, etc.
3	PDE of order two and higher with constant coefficient, Classification of PDE, Reduction to canonical or Normal form, Riemann Method, Monges's method.
4	derivation of fundamental solution, Uniqueness of solution, Dirichlet's principle, Method of separation of variables.
5	D'Alembert's solution, Uniqueness of solution, Method of separation of variables.

4.	**Books Recommended**
1	M. P. Coleman, "An Introduction to Partial Differential Equations with MATLAB", 2 nd Edition, CRC Press, Boca Raton,2013.
2	P. Prasad and R. Ravindran, "Partial Differential Equations", 3rd Edition, New Age International Publishers,New Delhi,India, 2022.
3	I. N. Sneddon, "Elements of Partial Differential Equations", McGraw-Hill,New York, 2013.
4	L. C. Evans, "Partial Differential Equations", 2nd Edition, American Mathematical Society, 2010.
5	T. Amarnath, "An Elementary Course in Partial Differential Equations", 2nd Edition, Narosa Publications, New Delhi,2011.

M.Sc. IV (Mathematics) Semester – VIII CALCULUS OF VARIATIONS & INTEGRAL EQUATIONS MA 432	Scheme	L	T	P	Credit
		3	1	0	04

1.	**Course Outcomes (COs):** **At the end of the course, students will be able to**
CO1	formulate variational problems and analyse them to deduce key properties of system behaviour.
CO2	explain the extremals of the BVP's through variational approach.
CO3	interpret the difference between Volterra and Fredholm integral equations.
CO4	solve IVP's and BVP's through integral equation approach.
CO5	solve integral equations through different analytical and numerical approach.

2.	**Syllabus**	
	INTRODUCTION TO CALCULUS OF VARIATIONS	**(09 Hours)**
	Maxima and minima, Natural boundary conditions and transition conditions, Variational notation, constraints and Lagrange multipliers, Hamilton's principle, Lagrange's equations, Constraints in dynamical systems.	
	VARIATIONAL PROBLEMS	**(09 Hours)**
	The Euler-Lagrange Equation, Minimum Surface of revolution, Geodesic, The Brachistochrone, Several dependent variables, Parametric representation, Undetermined end points, Brachistochrone from a given curve to a fixed point.	
	ISOPERIMETRIC PROBLEMS	**(09 Hours)**
	The simple isoperimetric problem, Direct Extension, Problem of the maximum enclosed area, Moving boundaries and transversality condition, Essential and Suppressible boundary conditions, Variational problems for deformable bodies, useful transformations, Rayleigh-Ritz method, Kantorovich method.	
	INTEGRAL EQUATIONS	**(10 Hours)**
	Linear Integral Equations, Eigen values and Eigen functions, The Green's function, Linear equations in cause and effect, The influence function, Fredholm equations with separable kernels, Hilbert Schmidt theory, Volterra Integral equation, Solution by Resolvent kernel, Method of successive approximations, The Neumann series, Fredholm theory, Singular Integral Equations.	
	APPROXIMATION OF INTEGRAL EQUATIONS	**(08 Hours)**

	Iterative approximations to characteristic functions, Approximations of Fredholm equations by sets of algebraic equations, Approximate method of undermined coefficients, the method of collocation, the method of weighting functions, the method of least squares, Approximation of the kernel.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**
	(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)	

3.	**Tutorials will be based on**
1	Maxima and minima, Natural boundary conditions and transition conditions, Variational notation, constraints and Lagrange multipliers
2	Hamilton's principle, Lagrange's equations, Constraints in dynamical systems.
3	The Euler-Lagrange Equation, Minimum Surface of revolution, Geodesic, The Brachistochrone, Several dependent variables
4	Parametric representation, Undetermined end points, Brachistochrone from a given curve to a fixed point
5	The simple isoperimetric problem, Direct Extension, Problem of the maximum enclosed area, Moving boundaries and transversality condition, Essential and Suppressible boundary conditions
5	Variational problems for deformable bodies, useful transformations, Rayleigh-Ritz method, Kantorovich method
7	Linear Integral Equations, Eigen values and Eigen functions, The Green's function, Linear equations in cause and effect, The inuence function, Fredholm equations with separable kernels,Hilbert Schmidt theory
8	Volterra Integral equation, Solution by Resolvent kernel, Method of successive approximations, The Neumann series, Fredholm theory, Singular Integral Equations.
9	Iterative approximations to characteristic functions, Approximations of Fredholm equations by sets of algebraic equations
10	Approximate method of undermined coefficients, the method of collocation, the method of weighting functions, the method of least squares, Approximation of the kernel

4.	**Books Recommended**
1	F. B. Hildebrand, Methods of Applied Mathematics, Dover Publications,New York, 2012

2	A. M. Wazwaz, “A First Course in Integral Equations”, 2nd Edition, World Scientific Publishing Company,Singapore,2015.
3	M. R. Seikh and P. K. Nayak, “Integral Equations and Calculus of Variations”, 2nd Edition, Alpha Science International Ltd., Oxford,UK,2020.
4	M. V. Klibanov, Inverse Problems and Carleman Estimates: Global Uniqueness and Approximation, De Gruyter, Berlin,Germany,2021.
5	R. P. Kanwal, Linear Integral Equations: Theory and Techniques, 3rd Edition, Birkhäuser,Boston, 2013.

5.	**Additional Books:**
1.	I. N. Sneddon, “Mixed Boundary Value Problems in Potential Theory”, 1st Edition, North Holland,Amsterdam,Netherlands,1966.
2.	A. J. Jerry, “Introduction to Integral Equations with Applications”, 2nd Edition, Wiley Publishers,Hoboken, 1999.
3.	R. Weinstock, “Calculus of Variation with Applications to Physics and Engineering”, Dover Publications,Mineola,New York,1974.
4.	R. P. Kanwal, “Generalized Functions: Theory and Applications”, 3rd Edition, Academic Press, New York, 2004.

M.Sc. IV (Mathematics) SOBOLEV SPACE (ELECTIVE) MA451	Scheme	L	T	P	Credit
		3	1	0	04

1.	**Course Outcomes (COs):** **At the end of the course, students will be able to**
CO1	explain the concept of distribution
CO2	demonstrate L^1, L^2 and L^p Fourier transform
CO3	Explain the $W_{\infty}^{1,p}(\Omega)$, $W^{1,p}(\Omega)$ and $H^1(\Omega)$spaces and their properties
CO4	explain the imbedding theorem and weighted space
CO5	discuss various inequalities in Sobolev space

2.	**Syllabus**	
	DISTRIBUTION	**(04 Hours)**
	Test function spaces and distributions, Convergence distribution derivatives.	
	FOURIER TRANSFORM	**(07 Hours)**
	L^1 -Fourier transform, Fourier transform of a Gaussian, L^2 -Fourier transform, Inversion formula, L^p -Fourier transform, Convolution.	
	SOBOLEV SPACE	**(09 Hours)**
	The spaces $W_{\infty}^{1,p}(\Omega)$and $W^{1,p}(\Omega)$,Their simple characteristic properties, Density result, Min and Max of $W^{1,p}$ functions, The space $H^1(\Omega)$.	
	IMBEDDING THEOREM	**(06 Hours)**
	Continuous and compact imbedding of Sobolev spaces into Lebesgue spaces, Sobolev imbedding theorem, Rellich-Kondrasov theorem	
	WEIGHTED SPACE	**(09 Hours)**
	Definition, Motivation, Examples of practical importance, Special weights of power type, General weights, Weighted Lebesgue space $P(\Omega,\sigma)$, weighted Sobolev spaces $W^{k,p}(\Omega,\sigma)$, $W_{0}^{\Omega,p}$ and their properties.	
	INEQUALITIES	**(10 Hours)**
	Methods of local co-ordinates, The classes C^o, $C^{0,k}$, Holder's condition, Partition of unity, The class K(x0) including cone property. Hardy inequality, Jensen's inequality, Young's inequality, Hardy-Littlewood-Sobolev inequality, Sobolev inequality and its various versions.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**
	(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)	

3.	**Tutorials will be based on**
1	distributions.
2	Fourier transform.
3	Sobolev space.
4	imbedding theorem, weighted spaces etc.
5	inequalities.

4.	**Books Recommended**
1	Giovanni Leoni, A First Course in Sobolev Spaces, 2nd Edition, American Mathematical Society,Providence,2017.
2.	Barbara MacCluer, Elementary Functional Analysis, Graduate Texts in Mathematics, Springer,New York, 2010.
3.	Vladimir Maz'ya, Sobolev Spaces with Applications to Elliptic Partial Differential Equations, 2nd Edition, Springer,Berlin, Germany, 2011.
4.	J. J. Duistermaat and J. A. C. Kolk, Distributions: Theory and Applications, Springer,Berlin,Germany 2010.
5.	G. Leoni, "A First Course in Sobolev Spaces," American Mathematical Society,Providence, 2017.

5.	**Additional Books:**
1.	S. Kesavan, "Topics in Functional Analysis and Applications," John Wiley & Sons Ltd.,Chichester, 1989.
2.	A. Kufner, "Weighted Sobolev Spaces," John Wiley & Sons Ltd., New York, 1985.
3.	R. S. Pathak, "A Course in Distribution Theory and Applications," Narosa Publication House, New Delhi,2001.

M.Sc. IV (Mathematics) Semester – VIII ADVANCED MATHEMATICAL METHODS-II (ELECTIVE) MA 452	Scheme	L	T	P	Credit
		3	1	0	04

1.	**Course Outcomes (COs):** **At the end of the course, students will be able to**
CO1	demonstrate a common framework that distinguishes various multivariate analytic methods
CO2	to solve a mathematical problem by analytic methods
CO3	To express accuracy, efficiency and convergence properties
CO4	To implement mathematical method to get reliable oncome and
CO5	To relate with theory to solve various problems related to engineering and sciences
CO6	To deal mathematical problem by optimal analytic technique's behavior

2.	**Syllabus**	
	Mathematical Preliminaries	**(03 Hours)**
	Introduction. Motivation and purpose, Characteristic of homotropy analysis method.	
	Some analytic techniques	**(09 Hours)**
	Perturbation method, Lyapunov's artificial small parameter method, Adomian's decomposition method, The δ-expansion method	
	Homotopy analysis Method	**(09 Hours)**
	Zero-order deformation equation, High-order deformation equation, Convergence theorem, Some fundamental rules, Solution expressions, The role of the auxiliary parameter, Homotopy-Pad́ e method, systematic description for general nonlinear problems	
	Relationship to Euler transform	**(07 Hours)**
	Introduction, Generalized Taylor series, Homotopy transform, Relation between homotopy analysis method and Euler transform	
	Solution of nonlinear problems	**(10 Hours)**

	Homotopy analysis solution, Zero-order deformation equation, High-order deformation equation, Convergence theorem, Nonlinear boundary-value problems with multiple solutions, Introduction, Brief mathematical formulas, Nonlinear diffusion-reaction model, A three-point nonlinear boundary-value problem, Channel flows with multiple solutions	
	Optimal homotopy analysis method	**(07 Hours)**
	Introduction, Basic ideas, Different types of optimal methods, Systematic description and discussions.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**

(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)

3.	**Tutorials will be based on**
1	Perturbation method, Lyapunov's artificial small parameter method, Adomian's decomposition method, The δ-expansion method.
2	Zero-order deformation equation, High-order deformation equation, Convergence theorem, Some fundamental rules.
3	Homotopy transform, Relation between homotopy analysis method and Euler transform.
4	Nonlinear diffusion-reaction model, A three-point nonlinear boundary-value problem, Channel flows with multiple solutions.
5	Different types of optimal methods, Systematic description and discussions.

5.	**Books Recommended**
1	S.J. Liao, Advances in the Homotopy Analysis Method. World Scientific Publishing Company, Singapore, 2013.
2	S. I. Hayek, "Advanced Mathematical Methods in Science and Engineering", 2nd Edition, Chapman and Hall/CRC,Boca Raton, 2010.
3	S. Liao, Homotopy Analysis Method in Nonlinear Differential Equations, Springer, New York, 2012
4	Shijun Liao, "Beyond perturbation : introduction to homotopy analysis method", Champman& Hall/CRC, Boca Raton,2011.
5	S. Liao, "Homotopy Analysis Method in Nonlinear Differential Equations", SpringerVerlag Berlin Heidelberg, 2012.

M.Sc. IV (Mathematics) NATURAL LANGUAGE PROCESSING CS 461	Scheme	L	T		Credit
		3	1	0	04

1.	Course Outcomes (COs):
	At the end of the course, students will be able to
CO1	Explain basics principles of natural language processing.
CO2	apply machine learning techniques for NLP based different tasks.
CO3	perform statically analysis and classification, recognition using NLP knowledge acquired.
CO4	evaluate the performance of machine translation solutions through statistical parameters.
CO5	design efficient solution for parser, translator and different applications based on NLP for day to day usage.

2.	Syllabus	
	INTRODUCTION	**(04 Hours)**
	Human Languages, Language Models, Computational Linguistics , Ambiguity and Uncertainty in Language, Processing Paradigms; Phases in Natural Language Processing, Basic Terminology, Overview of Different Applications, Regular Expressions and Automata, Finite State Transducers and Morphology, Automata, Word Recognition, Lexicon, Morphology, Acquisition Models, Linguistics Resources, Introduction to Corpus, Elements in Balanced Corpus.	
	SYNTAX AND SYMANTICS	**(08 Hours)**
	Natural Language Grammars, Lexeme, Phonemes, Phrases and Idioms, Word Order, Tense, Probabilistic Models of Spelling, N-grams, Word Classes and Part of Speech Tagging using Maximum Entropy Models, Transformation Based Tagging (TBL), Context Free Grammars for English, Features and Unification, Lexicalized and Parsing, Treebanks, Language and Complexity, Representing Meaning, Semantic Analysis, Lexical Semantics, Word Sense Disambiguation.	
	PROBABILISTIC LANGUAGE MODELLING	**(08 Hours)**
	Statistical Inference, Hidden Markov Models, Probabilistic (weighted) Finite State Automata, Estimating the Probability of a Word, and Smoothing, Probabilistic Parsing, Generative Models of Language, Probabilistic Context Free Grammars, Probabilistic Parsing, Statistical Alignment and Machine Translation, Clustering, Text Categorization, Viterbi Algorithm for Finding Most Likely HMM Path.	
	PRAGMATICS	**(07 Hours)**
	Discourse, Dialogue and Conversational Agents, Natural Language Generation, Machine Translation, Dictionary Based Approaches, Reference Resolution, Algorithm for Pronoun Resolution, Text Coherence, Discourse Structure, Applications of NLP- Spell-Checking.	

	MACHINE TRANSLATION	**(09 Hours)**
	Probabilistic Models for Translating One to Another Language, Alignment, Translation, Language Generation, Expectation Maximization, Automatically Discovering Verb Subcategorization, Language Modelling Integrated into Social Network Analysis, Automatic Summarization, Question-Answering, Interactive Dialogue Systems.	
	ADVANCED TOPICS	**(09 Hours)**
	Summarization, Information Retrieval, Vector Space Model, Term Weighting, Homonymy, Polysemy, Synonymy, Improving User Queries, Document Classification, Sentence Segmentation, and Other Language Tasks, Automatically-Trained Email Spam Filter, Automatically Determining the Language, Speech Recognition.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**
	(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)	

3.	**Tutorials will be based on**
1	Human Languages, Language Models, Computational Linguistics, Ambiguity and Uncertainity in Language, Processing Paradims; etc.
2	natural language grammars, phrases and idioms, spelling, N-grams, context language and complexity, Representing Meaning, Semantic Analysis, etc .
3	statistical Inference, Markov Models, Automata, Estimating the Probability of a Word and smoothing, Probabilistic Parsing, Generative Models of Language, Probabilistic Contex Free Grammers, etc.
4	probabilistic models, Alignment, Translation, Language Generation, etc.
5	summarization, Information Retrieval, Vector Space Model, Term Weighting, Homonymy, Polysemy, Synonymy, Improving User Queries, etc.

4.	**Books Recommended**
1	D. Jurafsky and J. H. Martin, “Speech and Language Processing”, 3rd Edition, Pearson,Boston, 2025.
2	Y. Goldberg, Neural Network Methods for Natural Language Processing, Morgan & Claypool Publishers, San Rafael,2017.
3	J. Eisenstein, Introduction to Natural Language Processing, MIT Press,Cambridge, 2019.
4	T. Wolf et al., Transformers for Natural Language Processing: Build and Train State-of-the-Art Models, O'Reilly Media,Sebastopol, 2020.
5	Jacob Perkins, “Python Text Processing with NLTK 2.0 Cookbook”, Packt Publishing, Birmingham,UK,2010.

5. ADDITIONAL REFERENCE BOOKS	
1	A. Bharati, R. Sangal and V. Chaitanya, “Natural Language Processing: A Paninian Perspective“,PHI,New Delhi, India, 2000.
2	T. Siddiqui and U. S. Tiwary, “Natural Language Processing and Information Retrieval”, Oxford University Press, New Delhi, India,2008.
3	Floyd and Jain, “Digital Fundamentals”, Pearson Education,Boston, 2006.

M.Sc. IV (Mathematics) DATA ANALYTICS MA 453	Scheme	L	T	P	Credit
		3	1	0	04

1.	Course Outcomes (COs):
	At the end of the course, students will be able to
CO1	Explain types of data and various data science approaches
CO2	apply various data pre-processing and manipulation techniques including various distributed analysis paradigm using hadoop and other tools and perform advance statistical analysis to solve complex and large dataset problems.
CO3	analyse different large data like text data, stream data, graph data.
CO4	interpret and evaluate various large datasets by applying Data Mining techniques like clustering, filtering, factorization.
CO5	design the solution for the real life applications.

2.	Syllabus	
	INTRODUCTION	**(03 Hours)**
	Examples, Applications and results obtained using data science techniques, Overview of the data science process.	
	MANAGING LARGE SCALE DATA	**(03 Hours)**
	Types of data and data representations, Acquire data (E.G., Crawling), Process andparse data, Data manipulation, Data wrangling and Data cleaning.	
	PARADIGMS FOR DATA MANIPULATION, LARGE SCALE DATA SET	**(09 Hours)**
	MapReduce (Hadoop), Query large data sets in near real time with Pig and Hive, Moving from traditional warehouses to map reduce. Distributed databases, Distributed Hash Tables.	
	TEXT ANALYSIS	**(10 Hours)**
	Data flattening, Filtering and chunking, Feature scaling, Dimensionality reduction, Nonlinear factorization, Shingling of Documents, Locality Sensitive Hashing for Documents, Distance Measures, LSH Families for Other Distance Measures, Collaborative filtering.	
	MINING DATA STREAM	**(08 Hours)**
	Sampling data in a stream, Filtering streams, Counting distinct elements in a stream, Moments, Windows, Clustering for streams.	

	ADVANCED DATA ANALYSIS	**(12 Hours)**
	Graph visualization, Data summaries, Hypothesis testing, ML model-checking and comparison, Link analysis, Mining of graph, Frequent item sets analysis, High dimensional clustering, Hierarchical clustering, Recommendation systems.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**

3.	**Tutorials will be based on**
1	examples, Applications and results obtained using data science techniques.
2	Managing Large Scale data: Types of data and data representations, Acquire data(E.G., Crawling), Process and Parse data, Data Manipulation, Data Wrangling and Data cleaning.
3	Paradigms for Data Manipulation, Large scale Data set.
4	Text Analysis I: Data Flattening, Filtering and Chunking, Feature scaling, Dimensionality reduction, Nonlinear Factorization, Shingling of Documents,
5	Text Analysis II: Locality Sensitive Hashing for Documents, Distance Measures, LSH Families for other distance Measures, Collaborative filtering.
6	Mining Data Stream: Sampling data in a a stream, Filtering streams, Counting distinct elements in a stream, Moments, Windows, Clustering for streams.
7	Advanced Data Analysis.

4.	**Books Recommended**
1	Tom White, "*Hadoop:The De_nitive Guide*, 4th Edition, O' Reilly Media, Sebastopol,2015.
2	Anand Rajaraman and Je_rey David Ullman, " *Mining of Massive Datasets*", 2nd Edition, Cambridge University Press,Cambridge, 2014
3	Andrew Bruce and Peter Bruce, "*Practical Statistics for Data Scientists*, 1st Edition, O'Reilly Publishing House, Sebastopol,2017.
4	J. Joel Grus, "*Data Science from Scratch*", 1st Editiion, O'Reilly Media, Sebastopol,2015
5	Montgomery, C. Douglas, and George C. Runger, "*Applied Statistics and Probability for Engineers*", John Wiley & Sons, 7th Edition, Hoboken, 2018.

M.Sc. IV (Mathematics) MULTIOBJECTIVE OPTIMIZATION MA 454	Scheme	L	T	P	Credit
		3	1	0	04

1.	Course Outcomes (COs):
	At the end of the course, students will be able to
CO1	classify simple (single) objective and multi-objective optimization problems
CO2	formulate real world problems in multiobjective optimization.
CO3	solve multiobjective optimization problems by classical approaches
CO4	solve multiobjective optimization problems by Evolutionary approaches
CO5	explain the importance of multiobjective optimization in real life

2.	Syllabus	
	ELEMENTS OF MULTIOBJECTIVE DECISION MAKING PROBLEM	**(06 Hours)**
	Introduction, Multiobjective decisions making process, Judgment and the value system, Decision making unit and the decision makers, Objectives and attributes, Decision Situation, Symbolic representation of the multiobjective Decision's problem, Scale of measurement, Elementary decision analysis, types of decision problems, Choosing a decision rule, decision tree.	
	MULTIOBJECTIVE OPTIMIZATION	**(07 Hours)**
	Definition of Multiobjective optimization, Difference between Single and Multi objective optimization, Formation of multiobjective optimization problem, Pareto-optimality, Efficiency and dominance, Compromise Solution.	
	METHODS TO SOLVE MULTIOBJECTIVE OPTIMIZATION PROBLEMS	**(14 Hours)**
	Graphical method, Multiobjective simplex method, Goal programming method, " constraint method, weighted sum method, Fuzzy programming approach with linear, exponential and hyperbolic membership function.	
	EVOLUTIONARY APPROACHES	**(09 Hours)**
	Introduction to Evolutionary approaches, Difficulties with classical optimization algorithm, Genetic Algorithm for the solution of multiobjective optimization problem.	
	SELECTED MULTIOBJECTIVE OPTIMIZATION PROBLEMS	**(09 Hours)**
	Multiobjective transportation problems, Multiobjective solid transportation problems, Multiobjective assignment problems.	
	Tutorials will be based on the coverage of the above topics separately.	**(15 Hours)**
	(Total Contact periods / Hrs.: 45 Hrs. + 15 Hrs. = 60 Hrs.)	

3.	**Tutorials will be based on**
1	basics of Elements of Multiobjectives Decision Making Problem I: Multiobjective decisions making process, Judgement and the value system, Decision making unit and the decision makers, Objectives and attributes, Decision Situation
2	Elements of Multiobjectives Decision Making Problem II:: Symbolic representation of the multiobjective Decision problem, Scale of measurement, Elemen tary decision analysis, types of decision problems, Choosing a decision rule, decision tree
3	Multiobjective Optimization I: Classification between single and multiobjective optimization, Formation of multiobjective optimization problem
4	Multiobjective Optimization II: Petro-optimality, Efficiency and dominance, Compromise solution in multiobjective optimization
5	Methods to solve Multiobjective Optimization Problem I: Graphical methos, multiobjective simplex method, Goal Programming method, Constraint Method
6	Methods to solve Multiobjective Optimization Problem II: Weighted sum method, Fuzzy programming approach with linear, exponential and hyperbolic membership method
7	Evolutionary Approaches I: Solve multiobjective optimization problems by Evolutionary approaches
8	Evolutionary Approaches II: Genetic Algorithm for the solution of multiobjective optimization Problem
9	Selected Multiobjective Optimization Problem I: Multiobjective transportation problems, Multiobjective solid transportation problems
10	Selected Multiobjective Optimization Problem II: Multiobjective assignment problems

4.	**Books Recommended**
1	C. Vira, Y.H. Yacon, "*Multiobjective Decision Making Theory and Methodology*", Dover Publication Inc., New York, 2012
2	K. Deb, "*Multiobjective Optimization using Evolutionary Algorithms*", John Willey & Sons,Hoboken, 2010
3	Kaoru Tone, Data Envelopment Analysis and Its Applications to Management, 2nd Edition, World Scientific, Singapore,2017.
4	D. T. Luc, "Multiobjective Linear Programming an Introduction", Springer International Publishing Switzerland, 2016
5	Xin-She Yang, Nature-Inspired Optimization Algorithms, 2nd Edition, Elsevier, Amsterdam, Netherlands, 2020.