Applied Mathematics & Humanities Department
अनुप्रयुक्त गणित और मानविकी विभाग

Comprehensive Written Examination
(Mathematics)

A. General Instructions

1. Comprehensive written exam for Ph. D. students of Mathematics shall be a single paper test of 100 marks and 3 hours duration. The question paper shall be divided in two Parts: A & B.

2. Part A shall contain 20 Multiple Choice Questions (MCQs) covering the topics given in the Section A of the syllabus. Each question shall be of one mark with no negative marking. The total marks allocated to this part shall be 20 out of 100.

3. A candidate shall be required to answer the questions from any 2 units from Part B. Each unit shall contain questions of 50 marks. The questions shall be of subjective/analytic nature where a candidate is expected to apply the scientific knowledge to arrive at the solution to the given scientific problem.

B. Syllabus

Part A

Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum.

Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems.

Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem

Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, General theory of homogenous and non-homogeneous linear ODEs, variation of parameters,

Lagrange and Charpit methods for solving first order PDEs, Classification of second order PDEs,

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods.

Part-B

Unit-I

Nature and scope of Operations Research, Simplex Algorithm, Simplex Tableau, Two Phase Method, Big-M Method, Revised simplex method, Duality, Dual simplex method.

Change in the objective function, Change in the requirement vector, Addition of a variable, Addition of a constraint, Parametric analysis of cost and requirement vector.

Gomory's cutting plane algorithm, Gomory's mixed integer problem algorithm, A branch and bound algorithm

Mathematical Model for Transportation Problem, North-West Corner Method, Lest Cost Method, Vogel's Approximation Method, Test for optimality, Degeneracy in Transportation Problem, Variations in Transportation Problem.

Mathematical Model for Assignment Problem, Solution Method for Assignment Problem, Variations in Assignment Problem, Traveling Salesman Problem.

Geometric programming (both unconstrained and constrained)

Inventory models –Deterministic and Probabilistic (with and without lead time)

Basic Structures of queuing models, Poisson queues –M/M/1, M/M/C for finite and infinite queue length, Non-Poisson queue -M/G/1, Machine Maintenance (steady state).

Unit-II

Definition of multi objective optimization, Pareto optimality, Efficiency and dominance, compromise solution. Weighted global criterion Method, Weighted sum method, Lexicographic Method, Weighted min-max method, Goal programming method, Fuzzy programming approach, Genetic algorithm, Jaya algorithm. Numerical illustrations.

Multi objective problems: Transportation problem, Multi objective Transportation Problem, Assignment problem, Multi objective Assignment Problem, Scheduling problem, PERT and CPM problem, Multi objective PERT and CPM problem. Fuzzy Multi-objective problem

Unit-III

Lagrange & Hermite type interpolation, Interpolation error, Numerical differentiation based on undetermined coefficients.

Orthogonal Polynomial: (Chebyshev, Legendre, Jacobi polynomials): Definition, orthogonality properties, integration and differentiation of. Collocation methods for boundary value problems using orthogonal polynomials. Least square methods for boundary value problems. Multistep methods for initial value problems of order one and two.

Radial basis functions (RBFs), positive definite matrices and functions, positive definite and strictly positive definite radial basis functions, Complete monotone and multiple monotone functions. Interpolation using RBFs.

Unit-IV

Porosity, Capillary Properties, Saturation, Wettability, Measurement of Capillary Pressure in a Porous Medium, Method of Centrifuge, Mercury Injection (Purcell Method), Permeability, Darcy's Law, Definition and Units of Permeability, Measurements of Permeability, Klinkenberg Effect, Analogies between the Laws of Darcy, Ohm and Fourier, Filtration Velocity, Quadratic Equation of Filtration, Relative Permeability

Fundamental Equation of Filtration, Differential Form of the Darcy-Law, Anisotropic Porous Media, Equation of State, Incompressible Fluids, Low Compressibility Fluids, Formation Volume Factor, Ideal and Real Gases, Equation of continuity, Special Forms of the Equation of Filtration, Incompressible Fluids, Low Compressibility Fluids, Elastic Porous Media, Real and Ideal Gases, Boundary and Initial Conditions, Discontinuities in Porous Media, Schematic of the Filtration Equations

Steady State Filtration, Non-Steady State Filtration in Infinite Acting Systems, Dimensionless Variables, The Infinite Radial System with Constant Pressure at the Interior Boundary, Non-

Steady State Filtration in a Finite System, Non-Steady State Filtration in Linear System, The Principle of Superposition

The Equation of Two-Phase Filtration, Vertical Two-Phase Filtration of Incompressible Fluids, The BUCKLEY-LEVERETT Solution, The Welge-Method, Influence of Gravity and Capillary Force, Influence of Gravity, Influence of the Capillary Force, The Capillary End-Effect, Imbibition

The Mobility Ratio, Propagation of a Displacement Front, Linear Displacement, Displacement in an Inclined Layer, Supercritical Displacement

Unit-V

Definition of sequence and subsequence, Limit of a sequence, Convergent sequence, Divergent sequence, Bounded and Monotonic sequence, Operations on convergent and divergent sequences, $T-$Strong convergence of numerical Sequences, relationship between ordinary convergence and $T-$Strong convergence, Limit superior and limit inferior, Cauchy sequences, Summability of sequences, $(C,1)$, $(C,2)$ and (C,k) summability and relationship between them. Convergence and divergence, Series with nonnegative terms, Alternating series, Conditional convergence and absolute convergence, Rearrangements of series, Tests for absolute convergence, $(C,1)$ Summability of series, the class l^2, The Schwarz inequality, Hölder and Minkowski inequalities.

Normed spaces, Fundamental of Normed Spaces, Sequence spaces, L^p-spaces, Function spaces, Inner product spaces, Continuity of linear maps, Banach spaces
Definition of Fourier series, Formulation of convergence problem, Pointwise convergence of Fourier series, Uniform convergence, $T-$Strong convergence of Fourier series, The Gibbs Phenomenon, A divergent Fourier series, Termwise integration, Trigonometric vs. Fourier series, Termwise differentiation, Other kind of summability, Cesaro Summability, The $(C,1)$ Summability of Fourier series, Toeplitz summability, regularity of Toeplitz summability, Abel Summability, Abel's theorem, regularity of the Abel summability method, Composition of two summability methods, Moment sequences and Hausdorff matrices, The smoothing effect of $(C,1)$ summation, Weierstrass approximation theorem, The l^2 theory of Fourier series, , Orthonormal expansion in $l^2[a,b]$.

Unit-VI

sequences in a metric space, Complete metric space: Completion of a metric space, Baire category theorem, Compact metric Spaces. Continuity, Uniform continuity.

Contraction mappings, contractive mappings, fixed points, Banach contraction principle, applications of Banach contraction principle.

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Topological Spaces, Countability Axioms, Seperability, Homeomorphism.

Bounded linear operators, Inner product spaces, orthonormal sets, compact operator, Hilbert spaces.

Nonexpansive mappings, basic fixed points theorem for Nonexpansive mappings

Unit-VII

Picard's and Peano's Theorems, Gronwall's inequality, maximal interval of existence. Behaviour of solutions of higher order differential equations, Matrix exponential solution, Phase space analysis of critical points, Asymptotic behaviour of critical points, Green's function, Sturm comparison theorems and oscillations, Eigenvalue problems.

Vibrations and heat flow, Vibrating string and drumhead, Properties of heat flow, Stationary heat distribution, Physical Boundary conditions, The energy method for the heat equation, Properties of Heat equation, The maximum-minimum principle, Uniqueness, Stability, Solution and Invariance properties of Heat equation, Interpretation of the solution, Dirac delta function, Heat conduction on the half-line, Neumann, Dirchlit and mixed boundary conditions.

Variational problems of deformable bodies, useful transformations, Variational problem for Elastic plate, Rayleigh-Ritz method, Relations between differential and integral equations, The Green’s function, Linear Equations in cause and effect, Fredholm equations with separable kernels, Hilbert-Schmidt theory, Iterative methods for solving Integral equations of the second kind, The Neumann series, Fredholm theory, Singular Integral Equations, special devices. Iterative approximations to characteristic functions, Approximations of Fredholm equations by sets of algebraic equations, Approximate method of undermined coefficients, The method of collocation, The method of weighting functions, The method of least squares, Approximation of the kernel.

Unit-VIII

Fourier and Laplace transform and their basic properties. Special Functions of the Fractional Calculus, Gamma Function, Mittag-Leffler function, Fractional Derivatives and Integrals. Grunwald-Letnikov Fractional Derivatives. Riemann Liouville Fractional Derivatives. Some Other Approaches.

Geometric and Physical Interpretation of Fractional Integration and Fractional Differentiation. Sequential Fractional Derivatives. Left and Right Fractional Derivatives. Properties of Fractional Derivatives. Laplace Transforms of Fractional Derivatives. Fourier Transforms of Fractional Derivatives. Mellin Transform of Fractional Derivatives.

Linear Fractional Differential Equations. Fractional Differential Equation of a General Form. Existence and Uniqueness Theorem as a Method of Solution. Dependence of a Solution on Initial Conditions. The Laplace Transform Method. Standard Fractional Differential Equations. Sequential Fractional Differential Equations.

Unit-IX

Introduction, Hypergeometric function and generalized hypergeometric function, Integral representation, Differential properties of hypergeometric function, Confluent hypergeometric function and its integral representation.

Introduction to generating functions, Generating functions of the family of the form $G(2xt - t^2)$, $e^t\varphi(t)$ etc. with suitable examples (Bessel function, Legendre Polynomial, Hermite polynomial and Laguerre Polynomial), Boas and Buck type, Pure recurrence relations, Appell, Sheffer type of polynomial sets.

Introduction of orthogonality, moment functional, and, Existence of Orthogonal Polynomial Set, Orthogonality of Bessel Function, Legendre polynomial, Hermite polynomial and Laguerre Polynomial.The fundamental recurrence formula, Zeros, Gauss quadrature, Kernel polynomials, Symmetric moment functional, certain related recurrence relations.

Introduction to Basic Hyper geometric series, q-analogue of orthogonal polynomials, q-Gamma and q-Beta functions.

Unit-X

Convex sets and their properties, Convex and concave functions-Definitions and basic properties, Some fundamental theorems for convex functions.

The minimization and saddlepoint problems, Some basic results for minimization and local minimization problems, Sufficient optimality criteria, Necessary optimality criteria.

Differentiable convex and concave functions, differentiable strictly convex and concave functions, twice differentiable convex and concave functions, twice differentiable strictly convex and concave functions.

Minimization problems and the Fritz John and Kuhn-Tucker stationary point problems, Necessary and sufficient optimality criteria. Lagrange's theory, Duality theory, Search techniques - one variable and several variables.

Duality in nonlinear programming, duality in quadratic programming, duality in linear programming.

Quasiconvex and quasiconcave functions, strictly quasiconvex and strictly quasiconcave functions, Pseudoconvex and pseudoconcave functions, Some properties and relations between quasiconvex, strictly quasiconvex, pseudoconvex, convex and strictly convex functions.